Mehrsa Faramani marzieh.faramani@aalto.fi

Span and Linear Independence

Problem Sheet, Monday 7.08.2023 (Room U358)

Question 1: The Hilbert matrix is a specific type of matrix where the (i, j)-th entry is given by $1/(i+j-1)$. Consider the 4×4 Hilbert matrix, denoted by A:

$$A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{bmatrix}$$

Demonstrate that the rows of the matrix A are linearly independent.

Question 2: Given the following vectors in $\mathbb{R}^4$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 5 \\ 2 \\ 1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 4 \\ 6 \\ 2 \\ 0 \end{bmatrix},$$

Your challenge is to find a basis for the space spanned by these vectors.