

Vectors

Problem Sheet, Friday 28.07.2023 (Room U358)

Question 1. For vectors $\mathbf{a}$ and $\mathbf{b}$, it is given that $|\mathbf{a} + \mathbf{b}| = 5$ and $|\mathbf{a} - \mathbf{b}| = 3$. Compute the inner product $\mathbf{a} \cdot \mathbf{b}$.

Question 2. Consider three vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $\mathbb{R}^3$ defined as follows:

$$\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix}.$$

- Calculate the vector $\mathbf{u} \times \mathbf{v}$.
- Determine whether the vector $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{w}$.
- Find a vector that is orthogonal to both $\mathbf{u}$ and $\mathbf{w}$.

Question 3. Consider a parallelepiped in three-dimensional space with three adjacent edges defined by the vectors $\mathbf{a} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$, and $\mathbf{c} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$. Find the volume of the parallelepiped using the dot and cross products.

