

A = LU

Problem Sheet, Friday 04.08.2023 (Room U358)

Question 1: In the PLU decomposition of a 5 by 5 matrix A , the permutation matrix P is employed to rearrange the rows, resulting in row 1 becoming row 3, row 2 becoming row 4, row 4 becoming row 5, row 5 becoming row 2 and row 3 becoming row 1. To further analyse calculate the inverse of P .

Question 2: Find the LU factorisation of the matrix A .

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{bmatrix}$$

Question 3: A Vandermonde matrix is a matrix in which each row is a geometric progression of the form $(1, a, a^2, a^3, \dots)$. Vandermonde matrices are special matrices with intriguing properties that find applications in various mathematical disciplines. In this question, we will explore the LU factorization of a Vandermonde matrix. Consider the 4×4 Vandermonde matrix A :

$$\begin{bmatrix} 1 & a & a^2 & a^3 \\ 1 & b & b^2 & b^3 \\ 1 & c & c^2 & c^3 \\ 1 & d & d^2 & d^3 \end{bmatrix}.$$

In a LU factorization of matrix A , the L and U are as follows :

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & \frac{c-a}{b-a} & 1 & 0 \\ 1 & \frac{d-a}{b-a} & \frac{(d-a)(d-b)}{(c-b)(c-a)} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & a & a^2 & a^3 \\ 0 & b-a & b^2-a^2 & b^3-a^3 \\ 0 & 0 & (c-b)(c-a) & (c-b)(c-a)(c+b+a) \\ 0 & 0 & 0 & (d-c)(d-b)(d-a) \end{bmatrix}.$$

Use this factorization to solve the following linear system of equations :

$$\begin{cases} x + y + z + w = 5 \\ x + 2y + 4z + 8w = 19 \\ x - y + z - w = -3 \\ x - 2y + 4z - 8w = -25. \end{cases}$$